doi: 10.18323/2782-4039-2025-3-73-3

Study of rigidity and frequency response of an end mill on a vertical milling centre

Roman D. Voronov*, teacher of Chair "Equipment and Technologies of Machinery Production"
Dmitry A. Rastorguev[1], PhD (Engineering), Associate Professor,
assistant professor of Chair "Equipment and Technologies of Machinery Production"
Denis G. Levashkin[2], PhD (Engineering), Associate Professor,
assistant professor of Chair "Equipment and Technologies of Machinery Production"
Togliatti State University, Togliatti (Russia)

***E-mail:** smr.rom@yandex.ru

[1]ORCID: https://orcid.org/0000-0001-6298-1068
[2]ORCID: https://orcid.org/0009-0007-2704-4635

Received 21.04.2025 *Revised 11.08.2025* *Accepted 04.09.2025*

Abstract: The study covers the problem of early elimination of tool resonant vibrations through preliminary mathematical modelling. In particular, the problem is considered for the case of milling with an end mill on a vertical milling centre. The paper presents processed experimental data and results of mathematical modelling containing information on the rigidity of the FKC 4257 mill, its natural frequencies on the spectrum and vibration modes. The constructed finite element mathematical model covers the mill itself, the gripping collet and the collet chuck attachment. The model describes the static rigidity of the mill with an error of 2.2 %, and the position of its natural frequencies on the spectrum – with an error of about 7 % relative to the experimental results. By constructing the amplitude-frequency characteristic and conducting a modal analysis, it is shown that the first two vibration modes (80 and 112 Hz) are the most critical for the mill, both in terms of the amplitude of vibrations and in terms of their shape. The vibration shapes in the first modes are bending. During the modal analysis, the vibration shapes in the remaining modes are considered and estimated. To improve the convergence of the frequency analysis results, it is proposed to introduce the coefficient K_{k1}=0.9, which takes into account the lower rigidity of a real mill in comparison with an idealized mathematical model, when applying which the convergence is improved to 2.5 %. Thanks to the applied technique, it is possible to obtain reliable data on the frequency zones of instability used in practice to avoid resonance phenomena. In the future, based on such data, taking into account the correction factors, it is possible to train neural network models predicting the tool response under specific processing conditions and solving the inverse problem of selecting rational tool geometry for specific tasks.

Keywords: end mill; rigidity; modal analysis; frequency analysis; mathematical modelling; amplitude-frequency characteristic; resonance.

Acknowledgments: The study was carried out within the state contract for R&D No. 125011300177-8.

For citation: Voronov R.D., Rastorguev D.A., Levashkin D.G. Study of rigidity and frequency response of an end mill on a vertical milling centre. *Frontier Materials & Technologies*, 2025, no. 3, pp. 39–49. DOI: 10.18323/2782-4039-2025-3-73-3.

INTRODUCTION

During milling with end mills on large-sized high-rigidity milling machines, the limiting elastic deformations are the release of the cutting tool itself, its vibrations with a certain amplitude and frequency [1; 2]. In cases when the frequencies of these vibrations do not resonate with the frequencies of other components of the "machine – fixture – tool – part" system (hereinafter referred to as MFTP), there is only a deviation from the specified shaping [3; 4] to one degree or another, i.e. a decrease in the processing accuracy [5; 6]. However, it is widely known that when resonance phenomena occur during milling, vibrations of increased amplitude occur, which can lead to accelerated wear [7], tool breakage [8; 9] due to fatigue failure and microchipping of its cutting edges, as well as to failure of spindle bearings and other machine components [10; 11]. It should be noted that the above-mentioned negative consequences of the mill operation in the resonant or near-resonant frequency range are visible both when milling difficult-to-machine materials along complex spatial curved trajectories and when processing relatively pliable materials along simple flat trajectories. In particular, this is expressed in the unsatisfactory quality of the machined surfaces and the discrepancy between the obtained dimensions and shapes of the part and the initial machining tolerances [12].

In this regard, the ability to abstract from the resonant frequency zones in advance remains relevant for mechanical machining; for this purpose, the vibration component of the machining is often analyzed and the frequency response of the mills are constructed [13; 14]. To solve this problem, both empirical methods [15; 16], based on the readings of eddy current sensors, and mathematical modelling, including the finite element analysis method [17; 18], are widely used. Moreover, to reduce and prevent the influence of tool vibrations, the results of modal analysis [19; 20] can be used, which provides an idea of the vibration forms [21] on the frequency spectrum.

The great majority of studies of cutting tools, in particular end mills, do not contain comprehensive consideration of the object in terms of both modal and frequency analyses and, as a result, no corresponding mathematical models with confirmed convergence are presented. Among the studies provided, when considering end mills, there are estimates of both the modes and amplitudes of vibrations on the frequency range, but separately from each other. Consequently, it is difficult to make an unambiguous conclusion about the criticality of a particular tool vibration frequency for milling. Therefore, an approach is required that combines an assessment by both the amplitude criterion and the vibration mode criterion.

The purpose of this study is to develop and test a method for end-to-end mathematical modelling, including static, modal and frequency analyses, using the example of the FKC 4257 mill installed on the MILLSTAR LMV 800 vertical milling centre; in this case, the modelling results should correlate with the experimental data.

METHODS

The end mill – collet chuck pair consisting of an FKC 4257 end mill with a diameter of 10 mm and a BT-40-ER32-100 collet chuck with a corresponding collet was investigated. The material of the mill is high-speed R6M5 steel. Modelling was carried out using the finite element method with Femap with NX Nastran v11. A static force of 400 N was used for static analysis and a unit force to obtain frequency responses. The MatLab package was used to process and visualize the experimental data.

The experimental part of the study includes measuring the static rigidity of the mill, as well as obtaining its frequency response. To conduct the experiment, a Z-shaped compression sensor was attached to the thread on the one side of the vertical plate fixed in a vice inside the working volume of the LMV 800 centre, and a screw was installed on the other side to load the mill. An indicator was mounted in a magnetic stand fixed to the machine spindle body. Measurements were taken using a DEP1 portable electronic dynamometer.

During this experiment, a series of measurements were carried out with different forces applied to the mill – from 50 to 400 N. Direct measurement of deformation was carried out using a strain-gauge dynamometer with displacement control by a clock-type indicator with a measurement error of 0.01 mm (right side of Fig. 1).

The natural frequencies of the end mill on the LMV 800 machine were obtained using the pulse method – by striking a dynamometric hammer with recording the response with a Bentley Nevada eddy current sensor.

As will be shown below, it is the first two vibration modes that have the largest amplitudes, as well as the most dangerous natural vibration modes. In this regard, when considering the peaks in the frequency spectrum, we will make comparisons based on the first two peaks obtained experimentally.

For the mathematical model, the conditions of fixing (boundary conditions) and loading were set according to the experiment, taking into account the materials of the bodies and the nature of their contact.

Obviously, the limiting deformations will be observed at the maximum extension of the mill from the chuck, and the chuck itself will practically not undergo deformation. In this regard, a proportional finite element mesh for various bodies was set, which allows reducing the time required for analysis.

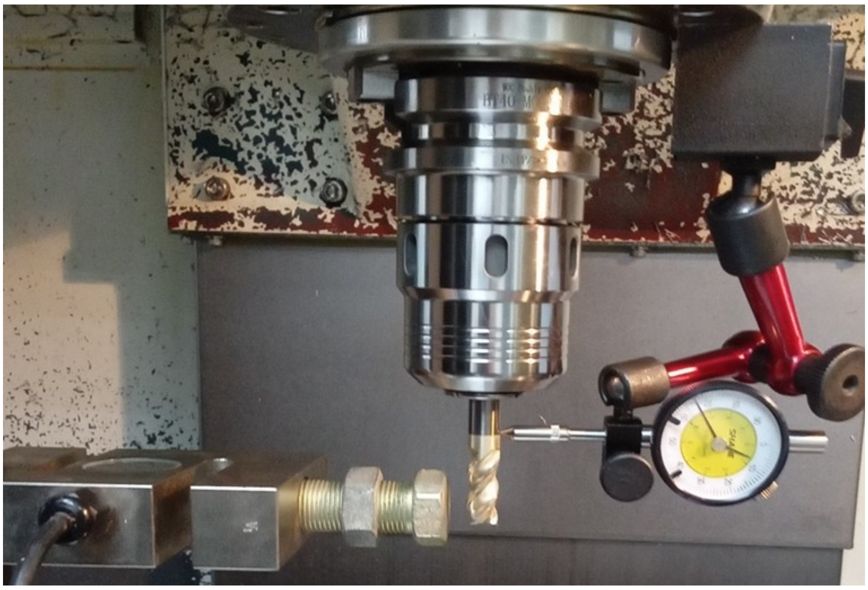

Fig. 1. *General view of the experimental installation inside the machine*
Рис. 1. *Общий вид экспериментальной установки внутри станка*

RESULTS

Since deformations occur deep in the elastic region, then, as expected, the compliance graph (Fig. 2) is a straight line with a certain coefficient responsible for its slope. Let us determine the rigidity at the measurement point (approximately 19 mm from the chuck) using the known relationship:

$$j = \frac{F}{\Delta};$$

$$j_{exp} = \frac{F}{\Delta} = \frac{400}{0.0513} \approx 7797.3 \frac{\mathrm{N}}{\mathrm{mm}}.$$

The vibration analyzer allowed obtaining the amplitude-frequency characteristic (hereinafter referred to as the AFC) of the mill with pronounced peaks (Fig. 3). The first two peaks are located at frequencies of 80 and 112 Hz, respectively. The parameters that were identified during measurements in these experiments are decisive in terms of forming accuracy through elastic deformations.

As can be seen from the static calculation (Fig. 4), under a load of 400 N, the greatest displacements are about 0.2 mm at the end of the mill. It is possible to visualize more clearly the distribution of deformations along the length of the mill using a graph. The graph (Fig. 5) shows the displacements along the *Z*-axis (the longitudinal axis of the mill), with the zero reference point being the point of zero mill extension from the chuck (the edge of the chuck). The obtained dependence, in accordance with expectations, is close to linear.

According to the calculation results, displacements of 0.05 mm obtained experimentally were recorded at

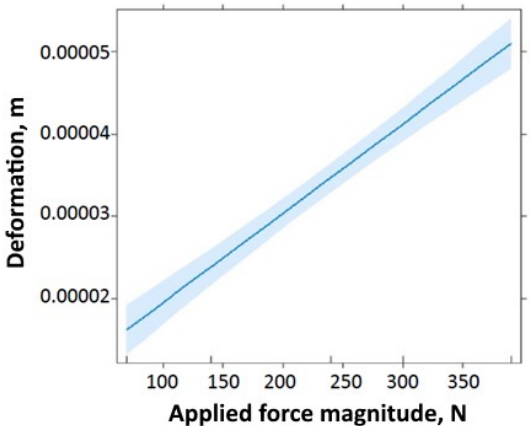

Fig. 2. *Experimental graph of the dependence of deformations (m) on the applied force (N). The highlighted area located along the graph line visualizes the 95 % confidence interval*
Рис. 2. *Экспериментальный график зависимости деформаций (м) от прилагаемой силы (Н). Выделенная область, расположенная вдоль линии графика, визуализирует 95%-й доверительный интервал*

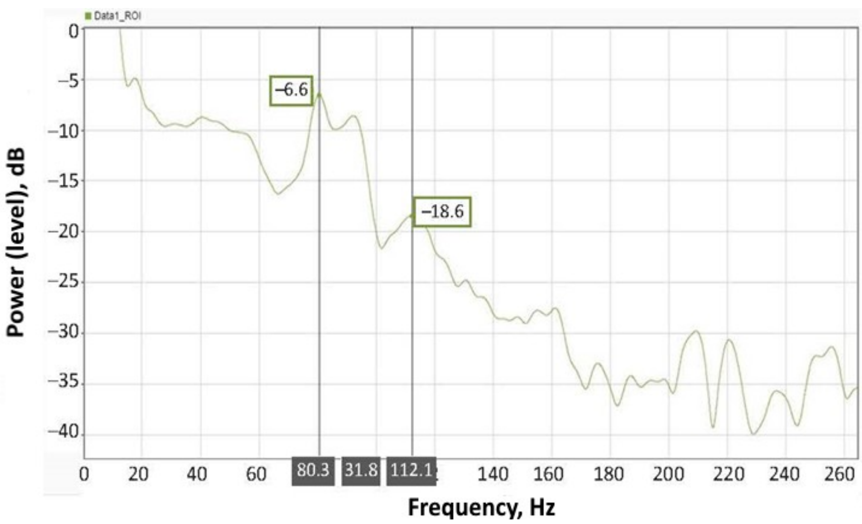

Fig. 3. *The amplitude-frequency characteristic obtained experimentally with the peaks marked*
Рис. 3. *Амплитудно-частотная характеристика, полученная экспериментально, с отмеченными пиками*

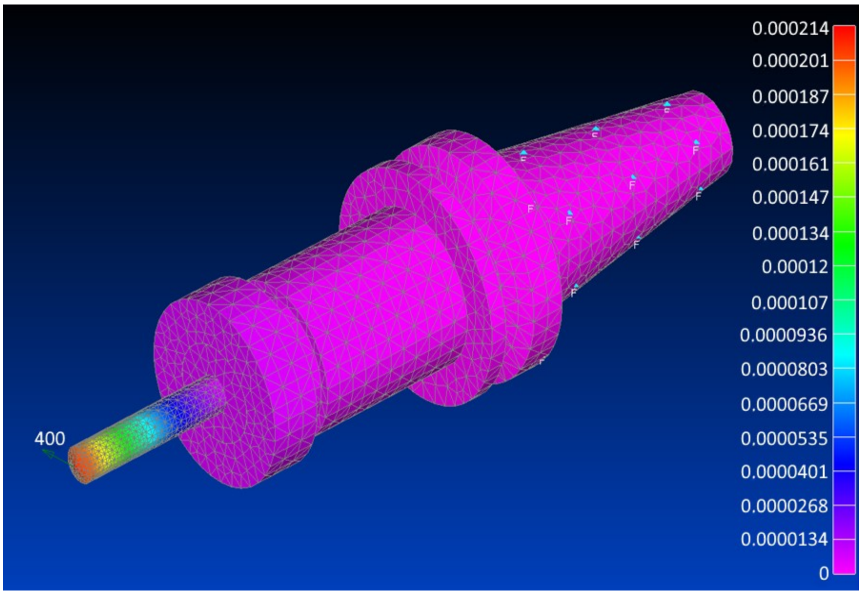

Fig. 4. *Result of static calculation for rigidity*
Рис. 4. *Результат статического расчета на жесткость*

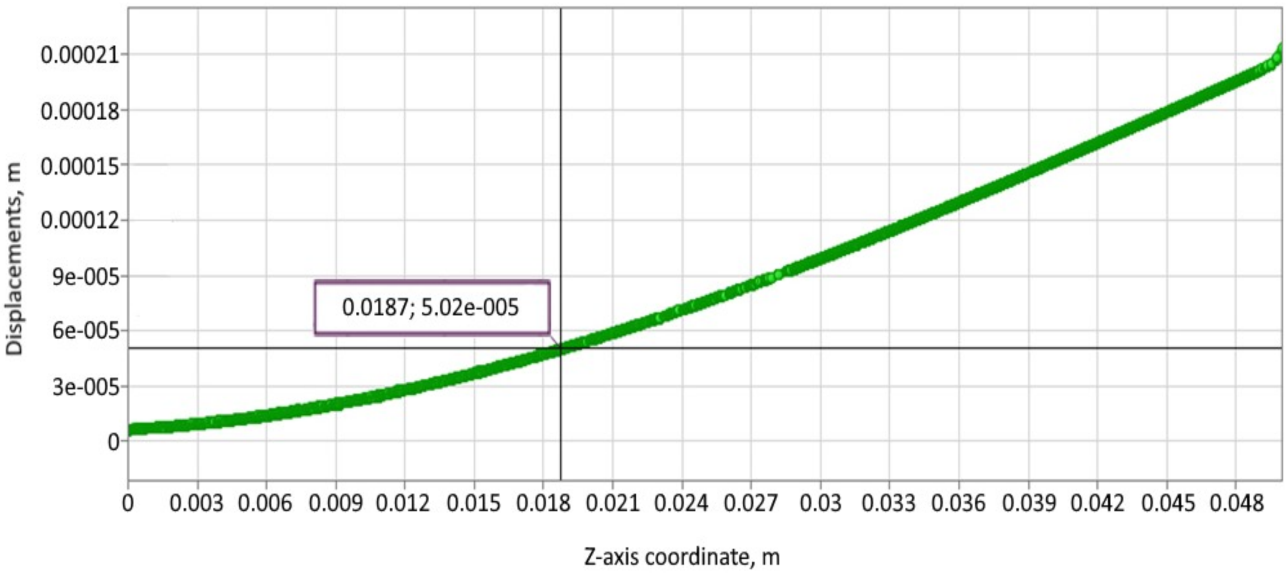

Fig. 5. *Graph of the dependence of displacements on the coordinate of the mill longitudinal axis (modelling)*
Рис. 5. *График зависимости перемещений от координаты продольной оси фрезы (моделирование)*

a distance of approximately 19 mm (Fig. 5) from the edge of the chuck (mill extension). This value coincides with the distance from the edge of the chuck to the measurement point during the experiment, which allows concluding that the created mathematical model for determining static rigidity is correct.

Then the rigidity at the measurement point is equal to

$$j_{model} = \frac{F}{\Delta} = \frac{400}{0.0502} \approx 7968 \, \frac{\text{N}}{\text{mm}} .$$

The obtained modelled rigidity value differs from that calculated from the experimental data by 2.2 %, which confirms the correlation.

To find the natural forms and frequencies of the structure, we will conduct a modal analysis for the same fixing conditions as in the case of a static calculation. The result of the modal analysis can be presented both as a set of modes (Fig. 6) and graphically on a frequency interval (Fig. 7), where the peaks of natural frequencies are clearly visible. On the graph (Fig. 7), the dimensionless coefficient, which does not show the amplitude of vibrations at resonant frequencies, is plotted along the ordinate axis, and the interval

Mode order number	Frequency, Hz
1	86.282
2	86.314
3	119.853
4	119.917
5	275.697
6	407.808
7	407.826
8	427.403
9	596.541
10	596.969

Fig. 6. *Vibration modes of the end mill – collet chuck pair*
Рис. 6. *Моды колебаний пары «концевая фреза – цанговый патрон»*

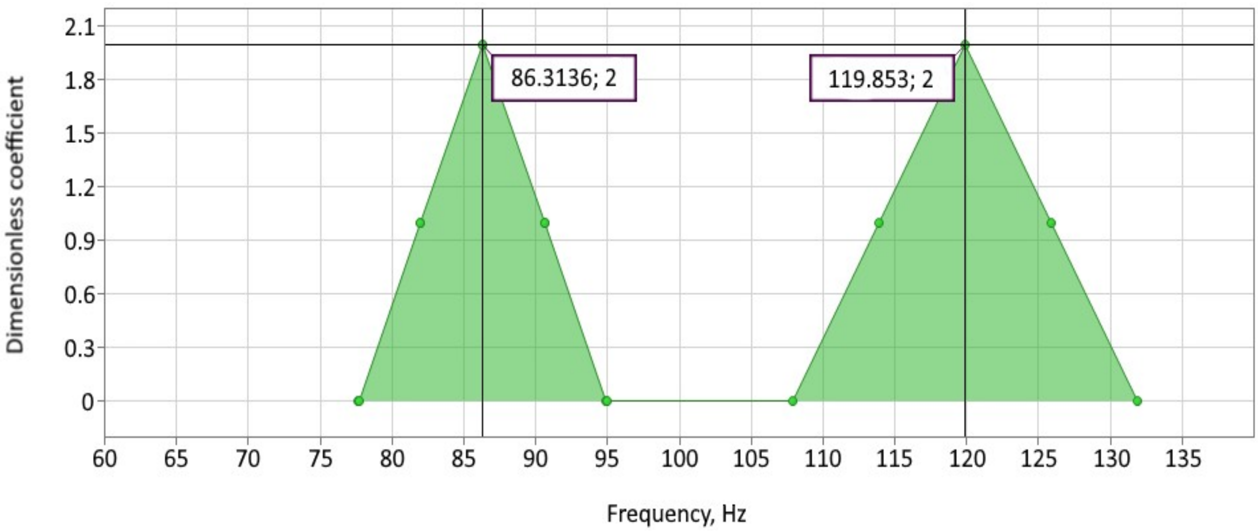

Fig. 7. *The location of the first two natural frequencies of the mill on the spectrum (modelling)*
Рис. 7. *Расположение первых двух собственных частот фрезы на спектре (моделирование)*

of the frequencies under consideration is plotted along the abscissa axis. The desired peaks are located at 86 and 120 Hz, respectively. When comparing the position of the peaks on the spectrum with the experimental data (Fig. 3), a discrepancy of approximately 6–8 Hz is noticeable.

As the modal analysis shows, in the first and second vibration modes (≈86 and 120 Hz), the natural form is bending, and in the instability zone there is only the mill itself, deviating from its axis to the side, which fully corresponds to the real direction of elastic deformations during milling with an end mill.

Note that modes 1 and 2, 3 and 4, 6 and 7, 9 and 10 have practically the same frequencies (Fig. 6), which is expressed graphically as a "merger" into a single peak of modes with serial numbers of 1 and 2, 3 and 4, respectively. Moreover, the natural forms of the mill at these frequencies are identical and they are bending (Fig. 8). Therefore, we will consider such close modes as a single natural frequency.

In the fifth mode of vibrations (≈276 Hz), the natural form is twisting around the longitudinal axis of the mill, and the collet chuck itself is mostly in the zone of instability (Fig. 9). The next, sixth and seventh modes (≈408 Hz), correspond to the form of spatial bending of the entire structure around the nodes indicated by the violet-coloured zones (Fig. 10). This form of vibrations is dangerous due to the presence of an inflection, where the form of vibrations changes its direction. In this case, both the mill itself and the chuck body are still in the vibration antinode zones. The natural form in the eighth mode (≈427 Hz) is compressive along the longitudinal axis of the tool, covering both the tool and the collet chuck (Fig. 11). The vibration antinode in this mode is concentrated on the mill.

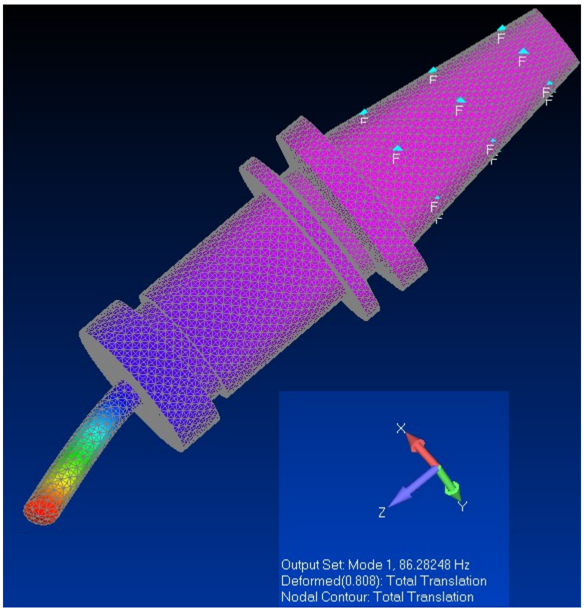

Fig. 8. *Vibration mode shape in the first mode for the case of a solid end mill*
Рис. 8. *Собственная форма колебаний на первой моде для случая цельной концевой фрезы*

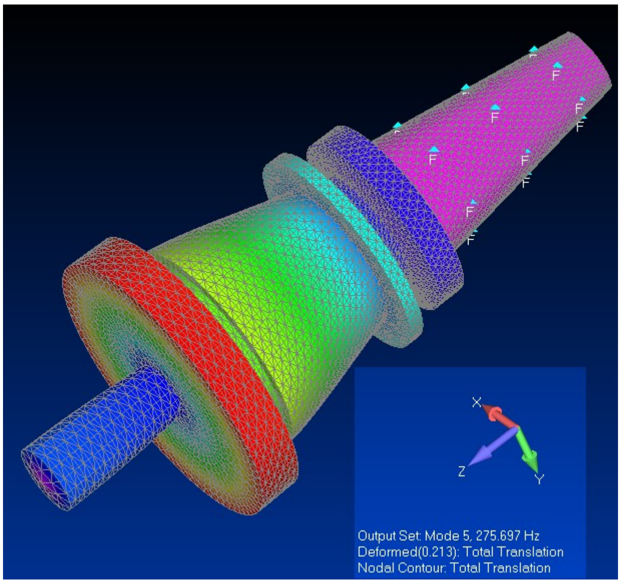

Fig. 9. *Vibration mode shape in the fifth mode for the case of a solid end mill*
Рис. 9. *Собственная форма колебаний на пятой моде для случая цельной концевой фрезы*

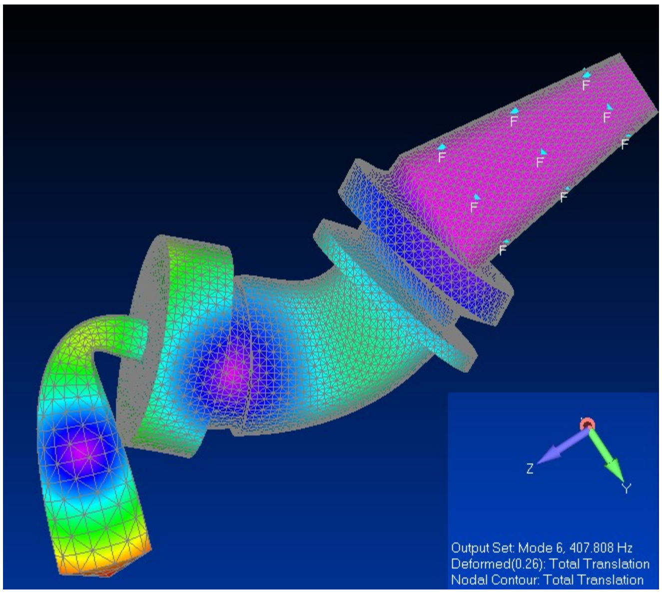

Fig. 10. *Vibration mode shape in the sixth mode for the case of a solid end mill*
Рис. 10. *Собственная форма колебаний на шестой моде для случая цельной концевой фрезы*

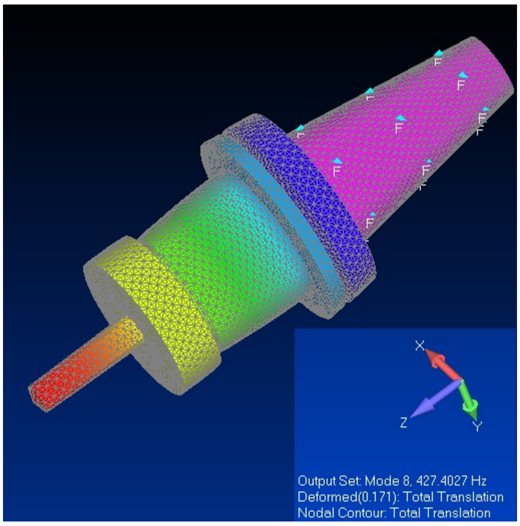

Fig. 11. *Vibration mode shape in the eighth mode for the case of a solid end mill*
Рис. 11. *Собственная форма колебаний на восьмой моде для случая цельной концевой фрезы*

The last, ninth and tenth vibration modes (≈596 Hz), correspond to the form of the spatial bending of the mill itself exclusively (Fig. 12), similar to the natural form in the sixth mode (Fig. 10). Here, the bend and nodes on the mill body are also traced.

Since the coefficient on the ordinate axis (Fig. 7) does not give any idea of the degree of criticality of a particular frequency according to the vibration amplitude criterion, it is necessary to carry out a frequency analysis next. Frequency analysis implies taking into account the damping of the vibrating system through the damping coefficient determined experimentally. For the case of a solid end mill, the logarithmic damping decrement was 0.06 according to empirical data obtained from the graph of the transient process of vibration

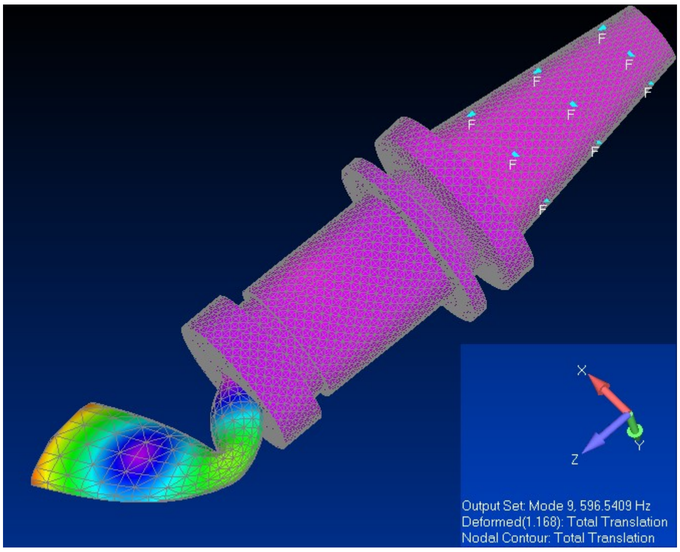

Fig. 12. *Vibration mode shape in the tenth mode for the case of a solid end mill*
Рис. 12. *Собственная форма колебаний на девятой моде для случая цельной концевой фрезы*

damping. Let us assume that this coefficient remains constant over the entire range of frequencies under consideration and is equal to the logarithmic damping decrement of vibrations.

The result of frequency analysis is the amplitude – frequency dependence in the relevant direction in space. In this case, we consider the transverse direction of the mill loading, i.e. the direction perpendicular to its axis. The frequency response (AFC) is obtained with a damping coefficient taken equal to 6 %. For a more demonstrative display of all amplitudes, we will use a logarithmic scale on the ordinate axis (Fig. 13).

According to the constructed AFC, it can be concluded that, according to the vibration amplitude criterion, the most dangerous are the first two natural frequencies of vibrations corresponding to frequencies of about 86 and 120 Hz (according to the simulation results). They are also the most dangerous according to the vibration form criterion, as shown by the modal analysis.

Thus, the results of static, modal and frequency analyses correlating with the experimental results are obtained.

DISCUSSION

The obtained results on the position of natural frequencies on the spectrum correlate with the experimental data within ≈7 %. Such a significant percentage of error is caused by the shift of the theoretical frequency response to the right in comparison with the empirical frequency response, which, in turn, occurs due to the idealization of the mathematical model. In particular, in the model subjected to finite element analysis, there are no mill chip grooves, which makes it more rigid and overstates the natural frequencies. Moreover, the type of frequency response and natural frequencies of a solid body is also affected by its geometric shape. Therefore, in the absence of wide and deep chip grooves, the result is somewhat distorted. In this regard, to improve convergence in such cases, it is possible to introduce a clarifying coefficient that will correct the rigidity of the mill and allow avoiding labour-intensive refinement of its model.

It is important to note that since the force action during the experiment was exerted on the mill, which is kinematically connected to the collet chuck, and the chuck, in turn, to the machine spindle, etc., the obtained frequency response is the resultant with respect to the machine itself and its rigidity. This contributed undoubtedly to the shape of the curve and the absolute values of the amplitudes [22]. This explains the inevitable external difference between the experimental frequency response and the frequency response obtained during the modelling.

For the case of a solid end mill, we introduce a dimensionless K_k coefficient, taking into account the presence of chip grooves in the section undergoing deformations. Obviously, the presence of chip grooves affects the movements along each of the coordinate axes differently, so the values of the coefficients should differ. Since for an axial tool in this work, the rigidity in the transverse direction (perpendicular to the tool axis) is considered, we define the corresponding coefficient K_{k1}=0.9. This coefficient value for each specific tool can be obtained as the ratio of the deformations of a solid cylindrical body (rod) and the considered real tool with chip grooves, all other things being equal.

When changing the rigidity of the model by 0.9 times, we obtain the following position of the peaks of the first two natural frequencies (Fig. 14). The first peak has shifted to the left to 82.2 Hz, the second one – to 114.3 Hz. Convergence with the experiment has been improved to 2.5 % (Fig. 3).

It is important to note that the introduction of a clarifying coefficient may not always be justified, since finding the peaks of natural frequencies is used in practice to avoid resonance phenomena, i.e. to separate operating modes and natural frequencies further from each other on the frequency spectrum. In this case, from a practical point of view, in some cases it is not necessary to know the position

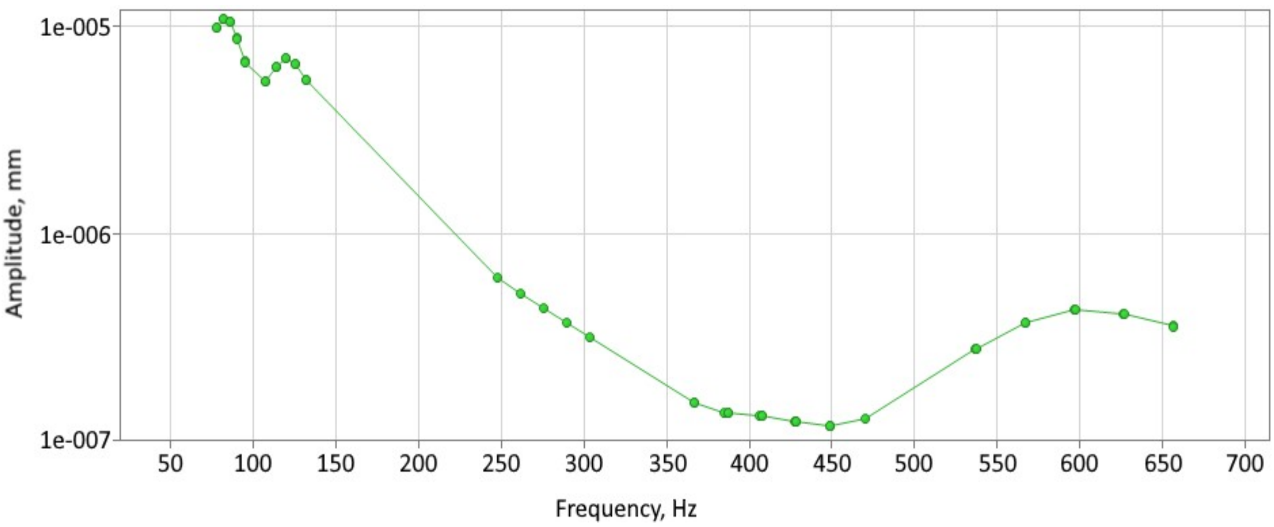

Fig. 13. *Amplitude-frequency characteristic in the cross direction with amplitude logarithmic scale (modelling)*
Рис. 13. *Амплитудно-частотная характеристика в поперечном направлении с логарифмической шкалой амплитуды (моделирование)*

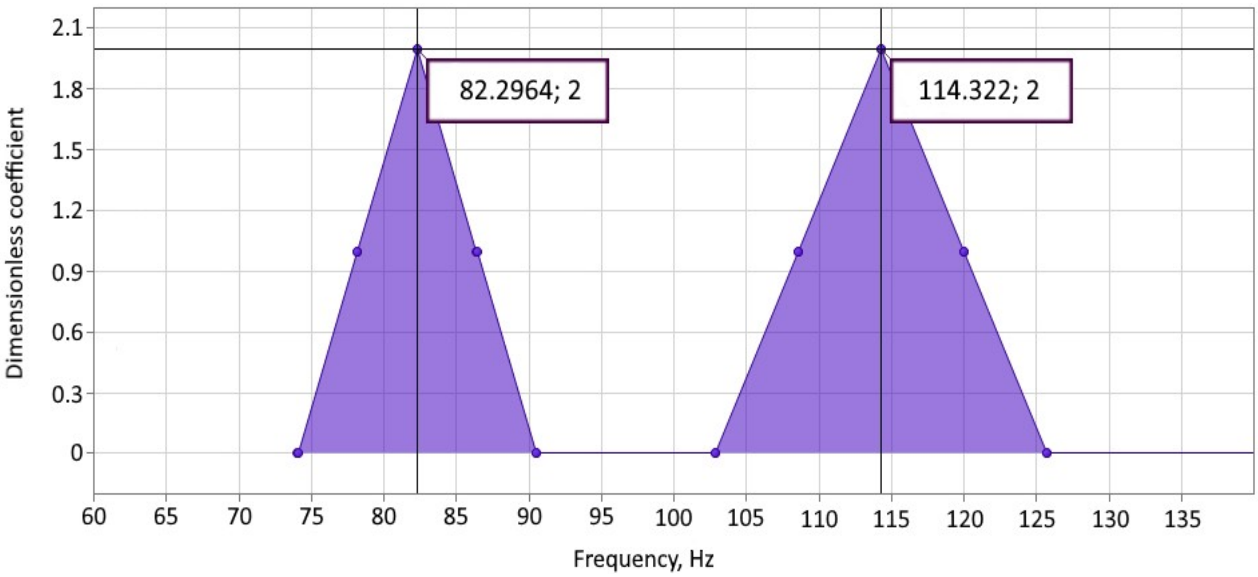

Fig. 14. *The location of the first two natural frequencies of the mill on the spectrum taking into account the correction factor (modelling)*
Рис. 14. *Расположение первых двух собственных частот фрезы на спектре с учетом поправочного коэффициента (моделирование)*

of the peak on the spectrum with an accuracy of 1–3 Hz. Knowing the presence of a peak in the range of about 10–15 Hz may be sufficient. The results of the static analysis (rigidity calculation) do not require clarification, initially, a convergence of 2.2 % is provided.

One of the directions for further development of the subject of this study is the expansion of the modelled system, i.e. the inclusion of a blank, a device, and the machine itself in the mathematical model, which will ultimately allow obtaining a model of a closed-loop "machine – fixture – tool – part" system. When maintaining the modelling methodology, it is expected to obtain convergence with the experiment, which will allow reflecting more fully and in detail the state of the technological processing system both in dynamics and in statics.

CONCLUSIONS

The presented rigidity modelling technique together with frequency analysis allows calculating force displacements and elastic deformations of the tool for its state at critical loading frequencies. It is the search for rigidity balance within the "machine – tool – part" system that forces us to evaluate the critical states of the tool, and then the machine, since it is the level of damping, rigidity and natural frequencies of the elements of the "machine – fixture – tool – part" (MFTP) system that determines the possibility and duration of the tool operation, which is the closing link (relatively low-rigidity and vulnerable) in the MFTP system.

The real response of the tool in the "machine – fixture – tool – part" system for individual cases, taking into account the geometry of the tool used, can be modelled with a convergence of 7 %. In this case, the results of mathematical modelling can be clarified using a coefficient that allows taking into account the chip grooves of the tool, which makes it possible to save time for the detailed development of a 3D model. When using the coefficient, the convergence of the peaks on the spectrum with the experiment reaches 2.5 %. Since the obtained calculated values of frequencies and deformations correlate with the experimental data, it can be stated that the goal of the work has been achieved.

Based on differential equations with a coefficient adopted in the first approximation, it becomes possible to train neural network models that will allow obtaining output data on the amplitude of tool deformation for any input data on the expected processing conditions. Moreover, this will allow the neural network and the user to formulate specific proposals for changing (improving) the input data to ensure optimal cutting conditions for a predetermined target function or for a whole set of target functions. In other words, it is possible to solve both the direct and inverse problems. With a sufficiently high level of training of such a neural network model, all the modal, frequency and rigidity characteristics of the tool of interest can be obtained from its output data without experiments or any modelling.

REFERENCES

1. Dovnar S.S., Yakimovich A.M. FEM analysis of the concept of a post-resonant portal multi-purpose machine. *Mashinostroenie: respublikanskiy mezhvedomstvennyy sbornik nauchnykh trudov*. Minsk, BNTU Publ., 2023. Vyp. 34, pp. 155–165.
2. Sazonov M.B., Zhidyaev A.N. Research on end mill vibration in machining of VT9 titanium alloy. *Vestnik of Samara University. Aerospace and Mechanical Engineering*, 2021, vol. 20, no. 4, pp. 89–99. DOI: 10.18287/2541-7533-2021-20-4-89-99.
3. Kalmykov V.V., Barkov A.V. Analytical and statistical assessment elastic deformation along the contour at milling. *Fundamental research*, 2016, no. 8-1, pp. 34–38. EDN: WHOECP.
4. Nurkenov A.Kh., Guzeev V.I., Batuev V.V., Nesteryuk E.V., Pavlov S.A. Experimental study of rigidity of a technological system on the basis of industrial robot KUKA KR 300 R2500 ULTRA. *Bulletin of the South Ural State University. Series "Mechanical engineering industry"*, 2022, vol. 22, no. 1, pp. 48–58. DOI: 10.14529/engin220104.
5. Kozlov A.M., Kuznetsov S.F., Kozlov A.A. Application of the active damper to reduce vibration when milling. *Actual problems in machine building*, 2021, vol. 8, no. 3-4, pp. 81–86. EDN: MQXJMI.
6. Markova E.V. Causes of vibration and methods of vibration extinguishing. *Izvestiya Tula State University. Tekhnicheskie nauki*, 2021, no. 5, pp. 428–432. EDN: CISDFC.
7. Yang Bin, Wang Min, Liu Zhihao, Che Changjia, Zan Tao, Gao Xiangsheng, Gao Peng. Tool wear process monitoring by damping behavior of cutting vibration for milling process. *Journal of Manufacturing Processes*, 2023, vol. 102, pp. 1069–1084. DOI: 10.1016/j.jmapro.2023.07.077.
8. Ponomarev B.B., Nguen V.D. Computing stresses in the wear zone of a radius end mill cutter. *BMSTU Journal of Mechanical Engineering*, 2025, no. 1, pp. 61–75. EDN: GFYZYA.
9. Lukyanov A.V., Aleynikov D.P., Kostin P.N. Investigation of oscillations in the forces of interaction of the milling cutter with the workpiece during passing and counter milling. *Modern Technologies. System analysis. Modeling*, 2022, no. 3, pp. 38–49. DOI: 10.26731/1813-9108.2022.3(75).38-49.
10. Aleynikov D.P., Lukyanov A.V., Kostin P.N. Cutting data effect on machine tool vibrational state and surface roughness when machining. *Systems. Methods. Technologies. Bratsk State University Scientific Journal*, 2021, no. 3, pp. 14–19. DOI: 10.18324/2077-5415-2021-3-14-19.
11. Kozlov A.M., Kiryushchenko E.V., Kozlov A.A. Expansion of technological capabilities of mobile milling equipment. *Bulletin of Voronezh State Technical University*, 2019, vol. 15, no. 4, pp. 140–145. DOI: 10.25987/VSTU.2019.15.4.022.
12. Gimadeev M.R., Li A.A., Berkun V.O., Stelmakov V.A. Experimental study of the dynamics of the machining process by ball-end mills. *Obrabotka metallov. Metal working and material science*, 2023, vol. 25, no. 1, pp. 44–56. DOI: 10.17212/1994-6309-2023-25.1-44-56.
13. Lukyanov A.V., Aleynikov D.P. Analysis of oscillations of cutting forces between a mill and a workpiece when increasing the spindle rotation speed. *Modern Technologies. System analysis. Modeling*, 2017, no. 4, pp. 70–82. EDN: YLXFFI.
14. Altintas Y., Lee P. Mechanics and dynamics of ball end milling. *Journal of manufacturing science and engineering*, 1998, vol. 120, no. 4, pp. 684–692. DOI: 10.1115/1.2830207.
15. Sazonov M.B., Zhidyaev A.N. Research on end mill vibration in machining of VT9 titanium alloy. *Vestnik of Samara University. Aerospace and Mechanical Engineering*, 2021, vol. 20, no. 4, pp. 89–99. DOI: 10.18287/2541-7533-2021-20-4-89-99.
16. Kravchenko K.Yu., Kugaevskiy S.S., Zhuravlev M.P., Elkind D.M. Natural frequencies estimation using operational modal analysis. *Bulletin of Perm National Research Polytechnic University. Mechanical engineering, materials science*, 2017, vol. 19, no. 2, pp. 21–35. DOI: 10.15593/2224-9877/2017.2.02.
17. Zhuravlev M.P., Kravchenko K.Yu., Elkind D.M. Vibration resistance at processing by end mills with variable pitch. *Bulletin of Bryansk State Technical University*, 2018, no. 8, pp. 14–24. DOI: 10.30987/article_5bb5e69727bb41.28471955.

18. Yatsun E.I., Zubkova O.S. Investigation of the tool system during boring and milling with a large departure. *Izvestiya Tula State University. Tekhnicheskie nauki*, 2024, no. 3, pp. 627–631. EDN: AXRLQN.
19. Matlygin G.V., Savilov A.V., Pyatykh A.S., Timofeev S.A. Study of the effect of cutting modes on output parameters under high-speed steel turn-milling. *Advanced Engineering Research (Rostov-on-Don)*, 2022, vol. 22, no. 2, pp. 99–106. DOI: 10.23947/2687-1653-2022-22-2-99-106.
20. Pyatykh A.S., Shaparev P.P. Investigating the impact of chucks on the stability of a milling process. *iPolytech Journal*, 2021, vol. 25, no. 5, pp. 549–558. DOI: 10.21285/1814-3520-2021-5-549-558.
21. Berthold J., Kolouch M., Regel J., Dix M. Identification of natural frequencies of machine tools during milling: comparison of the experimental modal analysis and the operational modal analysis. *Production Engineering*, 2024, vol. 18, no. 5, pp. 853–862. DOI: 10.1007/s11740-024-01270-6.
22. Berthold J., Regel J., Dix M., Drossel W.-G. Operational modal analysis used to identify dynamic behaviour of machine tools during milling. *Procedia CIRP*, 2023, vol. 118, pp. 128–133. DOI: 10.1016/j.procir.2023.06.023.

СПИСОК ЛИТЕРАТУРЫ

1. Довнар С.С., Якимович А.М. МКЭ-анализ концепции пострезонансного портального многоцелевого станка // Машиностроение: республиканский межведомственный сборник научных трудов. Минск: БНТУ, 2023. Вып. 34. С. 155–165.
2. Сазонов М.Б., Жидяев А.Н. Исследование вибраций концевых фрез при обработке титанового сплава ВТ9 // Вестник Самарского университета. Аэрокосмическая техника, технологии и машиностроение. 2021. Т. 20. № 4. С. 89–99. DOI: 10.18287/2541-7533-2021-20-4-89-99.
3. Калмыков В.В., Барков А.В. Аналитическое и статистическое оценивание упругих деформаций при фрезеровании по контуру // Фундаментальные исследования. 2016. № 8-1. С. 34–38. EDN: WHOECP.
4. Нуркенов А.Х., Гузеев В.И., Батуев В.В., Нестерюк Е.В., Павлов С.А. Экспериментальное исследование жесткости технологической системы на базе промышленного робота KUKA KR 300 R2500 ULTRA // Вестник Южно-Уральского государственного университета. Серия: Машиностроение. 2022. Т. 22. № 1. С. 48–58. DOI: 10.14529/engin220104.
5. Козлов А.М., Кузнецов С.Ф., Козлов А.А. Применение активного демпфера для снижения колебаний при фрезеровании // Актуальные проблемы в машиностроении. 2021. Т. 8. № 3-4. С. 81–86. EDN: MQXJMI.
6. Маркова Е.В. Причины возникновения вибраций и способы виброгашения // Известия Тульского государственного университета. Технические науки. 2021. № 5. С. 428–432. EDN: CISDFC.
7. Yang Bin, Wang Min, Liu Zhihao, Che Changjia, Zan Tao, Gao Xiangsheng, Gao Peng. Tool wear process monitoring by damping behavior of cutting vibration for milling process // Journal of Manufacturing Processes. 2023. Vol. 102. P. 1069–1084. DOI: 10.1016/j.jmapro.2023.07.077.
8. Пономарев Б.Б., Нгуен В.Д. Расчет напряжений в зоне износа режущей части концевой радиусной фрезы // Известия высших учебных заведений. Машиностроение. 2025. № 1. С. 61–75. EDN: GFYZYA.
9. Лукьянов А.В., Алейников Д.П., Костин П.Н. Исследование колебаний сил взаимодействия фрезы с заготовкой при попутном и встречном фрезеровании // Современные технологии. Системный анализ. Моделирование. 2022. № 3. С. 38–49. DOI: 10.26731/1813-9108.2022.3(75).38-49.
10. Алейников Д.П., Лукьянов А.В., Костин П.Н. Влияние параметров резания на вибрационное состояние станка и шероховатость обработанных поверхностей при механообработке // Системы. Методы. Технологии. 2021. № 3. С. 14–19. DOI: 10.18324/2077-5415-2021-3-14-19.
11. Козлов А.М., Кирющенко Е.В., Козлов А.А. Расширение технологических возможностей мобильного фрезерного оборудования // Вестник Воронежского государственного технического университета. 2019. Т. 15. № 4. С. 140–145. DOI: 10.25987/VSTU.2019.15.4.022.
12. Гимадеев М.Р., Ли А.А., Беркун В.О., Стельмаков В.А. Экспериментальное исследование динамики процесса механообработки концевыми сфероцилиндрическими фрезами // Обработка металлов (технология, оборудование, инструменты). 2023. Т. 25. № 1. С. 44–56. DOI: 10.17212/1994-6309-2023-25.1-44-56.
13. Лукьянов А.В., Алейников Д.П. Исследование колебаний сил взаимодействия фрезы с заготовкой при повышении скорости вращения шпинделя // Современные технологии. Системный анализ. Моделирование. 2017. № 4. С. 70–82. EDN: YLXFFI.
14. Altintas Y., Lee P. Mechanics and dynamics of ball end milling // Journal of manufacturing science and engineering. 1998. Vol. 120. № 4. P. 684–692. DOI: 10.1115/1.2830207.
15. Сазонов М.Б., Жидяев А.Н. Исследование вибраций концевых фрез при обработке титанового сплава ВТ9 // Вестник Самарского университета. Аэрокосмическая техника, технологии и машиностроение. 2021. Т. 20. № 4. С. 89–99. DOI: 10.18287/2541-7533-2021-20-4-89-99.
16. Кравченко К.Ю., Кугаевский С.С., Журавлев М.П., Элькинд Д.М. Операционный модальный анализ для определения собственных частот колебаний // Вестник Пермского национального исследовательского политехнического университета. Машиностроение, материаловедение. 2017. Т. 19. № 2. С. 21–35. DOI: 10.15593/2224-9877/2017.2.02.
17. Журавлев М.П., Кравченко К.Ю., Элькинд Д.М. Виброустойчивость при обработке торцевыми фрезами с переменным шагом // Вестник Брянского государственного технического университета. 2018. № 8. С. 14–24. DOI: 10.30987/article_5bb5e69727bb41.28471955.
18. Яцун Е.И., Зубкова О.С. Исследование инструментальной системы при растачивании и фрезеровании с большим вылетом // Известия Тульского государственного университета. Технические науки. 2024. № 3. С. 627–631. EDN: AXRLQN.

19. Матлыгин Г.В., Савилов А.В., Пятых А.С., Тимофеев С.А. Исследование влияния режимов резания на выходные параметры при точении фрезерованием быстрорежущей стали // Advanced Engineering Research (Rostov-on-Don). 2022. Т. 22. № 2. С. 99–106. DOI: 10.23947/2687-1653-2022-22-2-99-106.
20. Пятых А.С., Шапарев П.П. Исследование влияния зажимных патронов на устойчивость процесса фрезерования // iPolytech Journal. 2021. Т. 25. № 5. С. 549–558. DOI: 10.21285/1814-3520-2021-5-549-558.
21. Berthold J., Kolouch M., Regel J., Dix M. Identification of natural frequencies of machine tools during milling: comparison of the experimental modal analysis and the operational modal analysis // Production Engineering. 2024. Vol. 18. № 5. P. 853–862. DOI: 10.1007/s11740-024-01270-6.
22. Berthold J., Regel J., Dix M., Drossel W.-G. Operational modal analysis used to identify dynamic behaviour of machine tools during milling // Procedia CIRP. 2023. Vol. 118. P. 128–133. DOI: 10.1016/j.procir.2023.06.023.

УДК 621.9.022.2
doi: 10.18323/2782-4039-2025-3-73-3

Исследование жесткости и частотных характеристик концевой фрезы на вертикальном фрезерном центре

Воронов Роман Дмитриевич*, преподаватель
кафедры «Оборудование и технологии машиностроительного производства»
Расторгуев Дмитрий Александрович[1], кандидат технических наук, доцент,
доцент кафедры «Оборудование и технологии машиностроительного производства»
Левашкин Денис Геннадьевич[2], кандидат технических наук, доцент,
доцент кафедры «Оборудование и технологии машиностроительного производства»
Тольяттинский государственный университет, Тольятти (Россия)

***E-mail:** smr.rom@yandex.ru

[1]ORCID: https://orcid.org/0000-0001-6298-1068
[2]ORCID: https://orcid.org/0009-0007-2704-4635

Поступила в редакцию 21.04.2025 *Пересмотрена 11.08.2025* *Принята к публикации 04.09.2025*

Аннотация: Исследование посвящено проблеме заблаговременного исключения резонансных колебаний инструмента путем предварительного математического моделирования. В частности, проблема рассмотрена для случая процесса фрезерования концевой фрезой на вертикальном фрезерном центре. В работе приведены обработанные экспериментальные данные и результаты математического моделирования, содержащие сведения о жесткости фрезы ФКЦ 4257, ее собственных частотах на спектре и формах колебаний. Построенная конечно-элементная математическая модель охватывает саму фрезу, зажимную цангу и цанговый патрон. Модель описывает статическую жесткость фрезы с погрешностью 2,2 %, а положение ее собственных частот на спектре – с погрешностью около 7 % относительно результатов эксперимента. Посредством построения амплитудно-частотной характеристики и проведения модального анализа показано, что наиболее критичными для фрезы являются первые две моды колебаний (80 и 112 Гц), как по критерию величины амплитуды колебаний, так и по критерию их формы. Формы колебаний на первых модах являются изгибными. В рамках модального анализа рассмотрены и оценены формы колебаний на остальных модах. Для улучшения сходимости результатов частотного анализа предложено ввести коэффициент K_{k1}=0,9, учитывающий меньшую жесткость реальной фрезы в сравнении с идеализированной математической моделью, при применении которого сходимость улучшена до 2,5 %. Благодаря примененной методике можно получать достоверные данные о частотных зонах неустойчивости, используемые на практике для уходов от резонансных явлений. В перспективе на основе таких данных с учетом поправочных коэффициентов возможно обучение нейросетевых моделей, предсказывающих отклик инструмента при конкретных условиях обработки и решающих обратную задачу подбора рациональной геометрии инструмента под определенные задачи.

Ключевые слова: концевая фреза; жесткость; модальный анализ; частотный анализ; математическое моделирование; амплитудно-частотная характеристика; резонанс.

Благодарности: Исследование выполнено в рамках государственного контракта по НИОКТР № 125011300177-8.

Для цитирования: Воронов Р.Д., Расторгуев Д.А., Левашкин Д.Г. Исследование жесткости и частотных характеристик концевой фрезы на вертикальном фрезерном центре // Frontier Materials & Technologies. 2025. № 3. С. 39–49. DOI: 10.18323/2782-4039-2025-3-73-3.